Initial Condition and Cooling Time of Recombining Helium Plasma for Stationary VUV Lasing Action

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Calculated optimum conditions as to initial electron density and cooling rate of a recombining helium plasma for the production of VUV laser oscillation of the HeII 164 nm line are presented together with an explanation of the population inversion mechanism. For high initial plasma density ($\sim 10^{18} \, \mathrm{cm}^{-3}$) rapid cooling ($\tau_E \leq 10^{-13} \, \mathrm{sec}$) should produce a short pulse laser oscillation. For medium initial density ($\sim 10^{-16} \, \mathrm{cm}^{-3}$), stationary laser oscillation can be produced by much slower cooling ($\tau_E \sim 10^{-8} \, \mathrm{sec}$) but on cooling with $\tau_E > 10^{-7} \, \mathrm{sec}$ the oscillation can hardly be realized. For lower initial plasma density ($\leq 10^{15} \, \mathrm{cm}^{-3}$), laser oscillation is not possible because the population inversion obtained is not sufficient.

1. Introduction

The generation of stimulated emission in a recombining hydrogen plasma was first pointed out by Gudzenko and Shelepin [1, 2]. The idea of using a recombining multi Z hydrogenic ion plasma to create a short wavelength laser oscillation was proposed by Bohn [3] and developed in theoretical papers by Ali and Jones, Jones and Ali, Pert, and Suckewer and Fishman [4–8].

Recently, short pulse XUV lasers in the amplified spontaneous emission (ASE) obtained in a laser-produced plasma were reported by Matthews et al. and Suckewer et al. [9, 10]. The laser by Suckewer et al. was generated in a recombining plasma through radiation cooling. But stationary oscillation has not yet been observed in such short wave lasers.

Stationary VUV oscillation is of considerably practical interest but can probably only be obtained in a stationary recombining plasma flow, in which one can easily control the plasma parameters initial electron density $n_{\rm e0}$ and temperature $T_{\rm e0}$, cooling speed and final electron temperature $T_{\rm eend}$. An example of a stationary plasma flow is the magnetically confined plas-

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ma column called TPD-I plasma investigated by Otsuka et al. [11]. In the TPD-I recombining helium plasma, which was interacting with neutral helium gas without any expansion, Sato et al. observed a stationary population inversion between lower lying levels of He⁺, although the gain constant was too small for lasing [12]. This situation will be much improved if the initial electron density $n_{\rm e0}$ is considerably increased and the electron temperature $T_{\rm e}$ is further cooled down. In practical experiments, however, the cooling time is rather limited. Then it is very important to optimize $n_{\rm e0}$ for obtaining a sufficiently strong population inversion.

The main purpose of this paper is to present the optimum values of $n_{\rm e0}$ and cooling speed of $T_{\rm e}$ and to make clear the mechanism of the population inversion. Numerical calculation are performed for the HeII 160 nm line ($n=3 \rightarrow 2$ transition) as a representative case.

2. Simplified Model

We consider a helium plasma with a large ratio of length to radius. The plasma is confined by a magnetic field (cf. the TPD-I plasma). In this case, the particle confinement time is much longer than the

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electron temperature decay time. In our a simplified model $T_{\rm e}$ decreases with the decay time τ_E , and only the neighbourhood of the center axis of the plasma column is considered. Let the plasma flow v_0 along the center axis (z-axis) in the plasma column be constant. In a coordinate system fixed to the flow there is $\mathrm{d}/\mathrm{d}t = \partial/\partial t + v_0 \,\partial/\partial z$.

The plasma is composed of hydrogen-like He⁺ ions, fully stripped He²⁺ ions, He atoms, and electrons. The time derivative of the population density n_i (He) (cm⁻³) of the quantum state i of He is written as

$$\frac{\mathrm{d}n_i(\mathrm{He})}{\mathrm{d}t} = \sum_{j=1}^{49} a_{ij}(\mathrm{He}) \, n_i(\mathrm{He}) + \delta_i(\mathrm{He}),$$

$$1 \le i \le 49. \tag{1}$$

Here $a_{ij}(\text{He})$ and $\delta_i(\text{He})$ are given by the radiative transition probability $(A_{ij}(\text{He}), i < j)$ and by the rate coefficients for the electron collisional ionization $C_{ii}(\text{He})$ and recombination $K_i(\text{He})$, the electron collisional excitation $(C_{ij}(\text{He}), i < j)$ and deexcitation $(C_{ji}(\text{He}), i < j)$, and the radiative recombination $\beta_i(\text{He})$; i.e.,

$$\begin{split} a_{ij}(\text{He}) &= n_{\text{e}} \ C_{ji}(\text{He}), \quad j < i \,, \\ a_{ii}(\text{He}) &= -n_{\text{e}} \sum_{j=1}^{49} C_{ij}(\text{He}) - \sum_{j=1}^{i-1} A_{ji}(\text{He}), \\ a_{ij}(\text{He}) &= A_{ij}(\text{He}) + n_{\text{e}} \ C_{ji}(\text{He}), \quad j > i \,, \\ \delta_{i} &= n_{\text{e}} \ n_{1} (\text{He}^{+}) \left[n_{\text{e}} \ K_{i}(\text{He}) + \beta_{i}(\text{He}) \right], \end{split}$$

where $n_1(\text{He}^+)$ is the population density of the ground state of He^+ . These coefficients have been given by Fujimoto [13].

A corresponding equation is used for the population density $n_i(He^+)$ of quantum state i of He^+ :

$$\frac{dn_i(He^+)}{dt} = \sum_{j=1}^{20} a_{ij}(He^+) n_i(He^+) + \delta_i(He^+),$$

$$1 \le i \le 20,$$
(2)

where $a_{ij}(\text{He}^+)$ and $\delta_i(\text{He}^+)$, except for $a_{11}(\text{He}^+)$ and $\delta_1(\text{He}^+)$, are expressed in the same way as for He by using the rate coefficients for the helium ions given by Johnson [14]. Because the ionisation of He to the ground level of He⁺ and the inverse process must be taken into account, $a_{11}(\text{He}^+)$ and $\delta_1(\text{He}^+)$ are complicated:

$$a_{11}(\text{He}^+) = -n_e \sum_{j=1}^{20} C_{ij}(\text{He}^+)$$
$$-n_e \sum_{j=1}^{49} [K_i(\text{He}) n_e + \beta_i(\text{He})],$$

$$\delta_1(\text{He}^+) = n_e \, n(\text{He}^{2+}) \left[n_e \, K_i(\text{He}^+) + \beta_1(\text{He}^+) \right]$$

 $+ n_e \, \sum_{j=1}^{49} C_{jj}(\text{He}) \, n_j(\text{He}),$

where $n(\text{He}^{2+})$ is the stripped helium ion density, and the direct ionisation-excitation process from levels of He to excited levels of He^+ , and the inverse processes, are neglected.

The electric neutrality and the heavy particle conservation are assumed as follows:

$$n_e = 2 n(\text{He}^{2+}) + n(\text{He}^{+}),$$
 (3)

$$n_0 = n(\text{He}^{2+}) + n(\text{He}^+) + n(\text{He}),$$
 (4)

where n_0 is the total heavy particle density, which is assumed to be constant, and $n(\text{He}^{2+})$, $n(\text{He}^{+})$ and n(He) are the total densities of He^{2+} , He^{+} and He, respectively.

We assume that the plasma at t=0 is in a steady state with the electron density $n_{\rm e0}$ and temperature $T_{\rm e0}$. At t>0 we assume that $T_{\rm e}$ decreases to a final temperature $T_{\rm eend}$ with a decay time $\tau_{\rm E}$; the electron temperature changes as

$$T_{\rm e} = (T_{\rm e0} - T_{\rm eend}) \exp(-t/\tau_{\rm E}) + T_{\rm eend}.$$

We consider an optically thin plasma of the final temperature $T_{\text{cend}} = 0.2 \,\text{eV}$.

The integration of (1) and (2) was performed by using a high speed numerical method proposed by Tsuji et al. [15], using the Runge Kutta method.

3. Numerical Results and Discussion

The calculation was carried out for a steady state helium plasma in the initial electron temperature range $10 \, \text{eV} \le T_{\text{e}0} \le 200 \, \text{eV}$ and the initial electron density range $1.28 \times 10^{14} \, \text{cm}^{-3} \le n_{\text{e}0} \le 1.28 \times 10^{18} \, \text{cm}^{-3}$.

For $T_{e0} \ge 10 \text{ eV}$ one has $n_{e0} \approx 2 n_0 \text{ (He}^{2+})$, $n_0 \text{ (He}^{+})$ ≈ 0 and $n_0 \text{ (He)} \approx 0$, while for $T_{e0} \le 5 \text{ eV}$ one has $n_{e0} \approx n_0 \text{ (He}^{+})$, $n_0 \text{ (He}^{2+}) \approx 0$ and $n_0 \text{ (He)} \approx 0$.

First, we investigate the case of instantaneous cooling: $\tau_E = 0$.

Figure 1 shows the evolution of n_e , $n(\text{He}^2)$ and $n(\text{He}^+)$ for $T_{e0} = 200 \,\text{eV}$ and $n_{e0} = 1.28 \times 10^{14} \,\text{cm}^{-3}$, $1.28 \times 10^{16} \,\text{cm}^{-3}$ and $1.28 \times 10^{18} \,\text{cm}^{-3}$, respectively. Figure 2 shows the evolution of n_i/ω_i (i = 1, 2, 3) of He⁺, where i denotes the principal quantum number and ω_i is the statistical weight of the level i. Figure 3

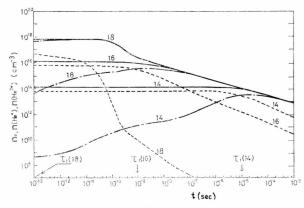


Fig. 1. $n_{\rm e}$ (solid curve), $n({\rm He^2}^+)$ (dashed curve) and $n({\rm He^+})$ (dash-dotted curve) vs. time for $T_{\rm e0}=200\,{\rm eV}$ and $\tau_{\rm E}=0$. "18" means the time history for $n_{\rm e0}=1.28\times 10^{18}\,{\rm cm^{-3}}$, etc. " $\tau_1(18)$ " means the recombination decay time of the fully stripped ion for $n_{\rm e0}=1.28\times 10^{18}\,{\rm cm^{-3}}$, etc.

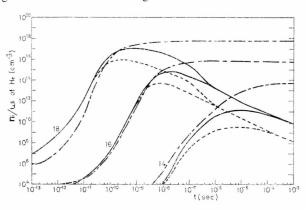


Fig. 3. The reduced population densities of He vs. time for $T_{\rm e0}=200\,{\rm eV}$ and $\tau_{\rm E}=0.~n_1/\omega_1$ (dash-dotted curve), n_2/ω_2 (solid curve) and n_3/ω_3 (dashed curve).

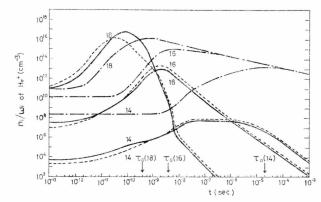


Fig. 2. Reduced population densities n_i/ω_i of He⁺ with principal quantum numbers i=1,2,3 vs. time for $T_{e0}=200\,\mathrm{eV}$ and $\tau_{\rm E}=0$. n_1/ω_1 (dash-dotted curve), n_2/ω_2 (solid curve) and n_3/ω_3 (dashed curve). "18" means the time history for $n_{e0}=1.28\times10^{18}\,\mathrm{cm}^{-3}$, etc. " $\tau_0(18)$ " means the time up to the maximum recombination of He⁺ for $n_{e0}=1.28\times10^{18}\,\mathrm{cm}^{-3}$, etc.

shows the evolution of n_i/ω_i (i = 1, 2, 3 corresponding to 1¹S, 2³S, 2¹S) of He.

In the very early time of the case of the high n_{e0} (1.28 × 10¹⁸ cm⁻³), the lower levels of He⁺ are not yet fully populated since the three body recombination process dominantly populates the upper levels. Then, there appears an orders of magnitude difference between $n(\text{He}^+)$ and $n_i(\text{He}^+)/\omega_i$ (i=1,2,3).

When $n_{\rm e0}$ is raised $(1.28 \times 10^{14} \, {\rm cm}^{-3}, 1.28 \times 10^{16} \, {\rm cm}^{-3})$ and $1.28 \times 10^{18} \, {\rm cm}^{-3})$, the decay time τ_1 of He²⁺ decreases $(10^{-5} \, {\rm sec}, 10^{-9} \, {\rm sec})$ and $10^{-13} \, {\rm sec})$ in

inverse proportion to the square of n_{e0} (see Fig. 1), and the time τ_0 to attain maximum recombination of He⁺ is reduced $(2\times 10^{-5}\,\mathrm{sec},\ 4\times 10^{-9}\,\mathrm{sec},\$ and $4\times 10^{-10}\,\mathrm{sec},\$ see Figure 2). Only in the case of extremely high initial electron density (1.28 $\times 10^{18}\,\mathrm{cm}^{-3}$) there is $\tau_1 \ll \tau_0$. For the lower initial electron densities the relation $\tau_1 = \tau_0$ is satisfied.

Note that in the case of $n_{e0} = 1.28 \times 10^{18} \,\mathrm{cm}^{-3}$, the collisional cascade process is quite predominant, and the recombination of He²⁺ to He⁺ proceeds abruptly until $t \sim \tau_0$. For the lower initial electron density, the rate of the collisional cascade processes decreases and the recombination to the lower energy levels of He⁺ is mainly due to radiative recombination. Then, the recombination to He, like that to He⁺ increases, hence $\tau_1 \approx \tau_0$. Thereafter $n(\text{He}^{2+})$ and $n(\text{He}^{+})$ decrease slowly due to the reduction of the electron density. For the low initial electron density $(1.28 \times 10^{14} \text{ cm}^{-3})$ it should be noted that a long time $\log (\tau_0 \approx \tau_1)$ $=10^{-5}$ sec) is indispensable for strong recombining and increasing of population densities after the instantaneous cooling. The relaxation time for recombination of the plasma will be approximated by a time τ_s with which the ground state of He reaches the steady state. The time τ_s , for example for the initial electron density 1.28×10^{18} cm⁻³, is 10^{-8} sec, as recognized from the time evolution of the ground state population density of He in Figure 3.

Figure 4 shows the time history of the over-population density $\Delta n_{32} = (n_3/\omega_3 - n_2/\omega_2)$. For the extremely high initial electron densities (1.28×10^{17})

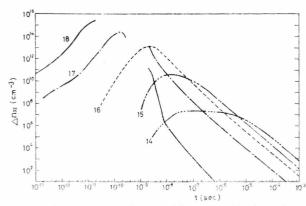


Fig. 4. The over population densities Δn_{32} of He⁺ vs. time for $T_{e0}=200\,\mathrm{eV}$ and $\tau_{\mathrm{E}}=0$. "18" means the time history for $n_{\mathrm{e}0}=1.28\times10^{18}\,\mathrm{cm}^{-3}$, etc.

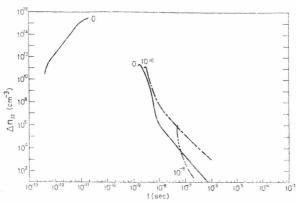


Fig. 5. The over population density Δn_{32} of He⁺ vs. time for $n_{e0}=1.28\times 10^{18}\,\mathrm{cm^{-3}}$ and $T_{e0}=30\,\mathrm{eV}$. " 10^{-8} " means the time history for $\tau_{\rm E}=10^{-8}\,\mathrm{sec}$, etc.

 $\sim 1.28 \times 10^{18} \,\mathrm{cm}^{-3}$), Δn_{32} increases in the early time of the recombination due to collisional cascade processes and then decreases abruptly with the decrease of He²⁺, followed by disappearance of the inverted population. When the spontaneous radiative transition process to the ground level becomes dominant for the depopulating mechansim of the level 2 due to the decrease of the electron density, the inversion between the levels is again produced, as shown by Bohn [3]. In the collisional cascade process, an extremely large maximum value of the over population density, $[\Delta n_{32}]_{\text{max}}$, is obtained, but Δn_{32} lasts only for a very short time (10^{-10} sec). In the low initial electron density plasmas, the levels 2 and 3 are not inversely populated in the early time because the three-body recombination process is not dominant. The inverted population is produced by the large spontaneous radiative transition process to the ground level.

The recombining plasma with the same initial electron density but various initial electron temperatures (over 10 eV) presents nearly the same evolution of n_e , $n(\text{He}^2^+)$, $n(\text{He}^+)$ and n(He) because of the same initial densities n_{e0} and $n_0(\text{He}^2^+)$. Note that Δn_{32} for an initial electron density plasma behaves almost in the same way for various initial electron temperatures (over $10 \, \text{eV}$).

Second, we show Δn_{32} during cooling with several decay times $\tau_{\rm E}$ when $T_{\rm e0}$ is fixed to be 30 eV. We will discuss the optimum values of $n_{\rm e0}$ and $\tau_{\rm E}$ to obtain the largest values of Δn_{32} .

For the extremely high value $n_{\rm e0} = 1.28 \times 10^{18} \, {\rm cm}^{-3}$ it is well understood that a large value of Δn_{32} , as realised in the instantaneous cooling plasma, is ex-

pected only for a rapid cooling time ($\tau_E \le 10^{-13}$ sec) because the decay time τ_1 of He²⁺ is 10^{-13} sec in the case of $\tau_E = 0$. If such a rapid cooling is realised, $[\Delta n_{32}]_{\rm max}$ of about 5×10^{15} cm⁻³ is obtained.

The gain per unit length, \varkappa , for the laser oscillation is expressed as $\varkappa = \omega_3 \, \sigma_{32} \, \Delta n_{32}$, where ω_3 is the statistical weight of the level 3 and σ_{32} is the cross section for the stimulated emission based on the broadening of the line by the Stark effect (see Suckewer and Fishman). Our calculation gives $\varkappa \sim 44 \, \mathrm{cm}^{-1}$ for this plasma with $\Delta n_{32} \approx 5 \times 10^{15} \, \mathrm{cm}^{-3}$, and the laser oscillation of the 164 nm line ($n=3 \to 2$ transition) will be easily realised. On the other hand, Δn_{32} is extremely reduced at slow cooling ($\tau_{\rm E} \sim 10^{-8} \, \mathrm{sec}$, cf. the curve shown by " 10^{-8} " in Figure 5) because the decay time $\tau_{\rm E} \sim 10^{-8} \, \mathrm{sec}$ is comparable to the time $\tau_{\rm S}$. In the extremely dense plasma ($\sim 10^{18} \, \mathrm{cm}^{-3}$), the laser oscillation is difficult to be realised on slow cooling ($\tau_{\rm E} \sim 10^{-8} \, \mathrm{sec}$).

The time history of Δn_{32} for the medium density plasma $(n_{\rm e0}=1.28\times 10^{16}\,{\rm cm}^{-3})$ is shown in Fig. 6 for various decay times $\tau_{\rm E}$. Δn_{32} starts increasing abruptly after a time lag of about $\tau_{\rm E}$ and reaches its maximum value $[\Delta n_{32}]_{\rm max}$, which decreases only slightly as $\tau_{\rm E}$ increases. $[\Delta n_{32}]_{\rm max}$ vs. $\tau_{\rm E}$ is shown in Fig. 7 for $n_{\rm e0}=1.28\times 10^{16}$ and $1.28\times 10^{15}\,{\rm cm}^{-3}$. $[\Delta n_{32}]_{\rm max}$ keeps almost constant for $\tau_{\rm E}<10^{-9}\,{\rm sec}$ for $n_{\rm e0}=1.28\times 10^{16}\,{\rm cm}^{-3}$ and $\tau_{\rm E}<10^{-8}\,{\rm sec}$ for $n_{\rm e0}=1.28\times 10^{15}\,{\rm cm}^{-3}$. This feature is favorable for the laser oscillation. We do not need to cool the plasma so rapidly and can realize a longer laser pulse. A practicable decay time $(\tau_{\rm E}=10^{-9}\sim 10^{-8}\,{\rm sec})$ can be obtained by the inelastic collision processes of elec-

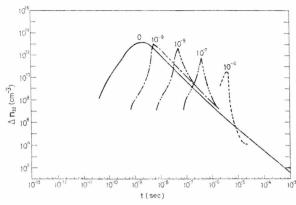


Fig. 6. The over population densities Δn_{32} of He⁺ vs. time for $\tau_{\rm E}\neq0$., $n_{\rm e0}=1.28\times10^{18}\,{\rm cm^{-3}}$ and $T_{\rm e0}=30\,{\rm eV}$. " 10^{-9} " means the time history for $\tau_{\rm E}=10^{-9}\,{\rm sec}$, etc.

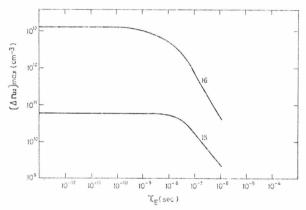


Fig. 7. The maximum over population density $[\Delta n_{32}]_{\rm max}$ of He⁺ vs. the decay time $\tau_{\rm E}$ for $T_{\rm e0}=30\,{\rm eV}$. "15" means $[\Delta n_{32}]_{\rm max}$ for $n_{\rm e0}=1.28\times10^{15}\,{\rm cm}^{-3}$.

trons with heavy particles as shown by Furukane and Oda, and Oda and Furukane [16, 17]. Nam et al. [18] have pointed out that radiation cooling leads to similar decay times. Suckewer and Fishman [8] show that for a carbon plasma due to radiation loss cooling without adiabatic expansion $\tau_E \approx 10$ nsec. Therefore it is expected that the cooling rate 10^{-8} sec can be achieved by adjusting the admixture of an efficiently radiative material (carbon) in the gain medium (helium) as shown by Nam et al. [18].

If $n_{\rm e0} = 1.28 \times 10^{16} \, {\rm cm}^{-3}$ and $\tau_{\rm E} = 10^{-8} \, {\rm sec}$, $[\Delta n_{32}]_{\rm max}$ of about $5 \times 10^{12} \, {\rm cm}^{-3}$ is obtained. We estimate \varkappa to be about $2 \, {\rm cm}^{-1}$ for the above recombining plasma with $\Delta n_{32} \sim 5 \times 10^{12} \, {\rm cm}^{-3}$. Since such a high gain can

be realised only in optically thin plasmas, let us consider a sheet plasma flow. If the sheet has a with of $l \ge 5$ cm the total gain $\varkappa l$ will be more than 10 in the transverse direction to the plasma flow. Then we can expect ASE laser oscillation of the 164 nm line in this plasma. Furthermore, stationary oscillation is possible because of the plasma flow. But the ASE laser oscillation can hardly be realized by cooling with the decay time $\tau_{\rm E} \ge 10^{-7}$ sec because $[\varDelta n_{32}]_{\rm max}$ is smaller than $8 \times 10^{11} \, {\rm cm}^{-3}$, that is, $\varkappa \le 0.3 \, {\rm cm}^{-1}$.

For the dilute density plasma $(n_{e0} = 1.28 \times 10^{15} \text{ cm}^{-3})$ $[\Delta n_{32}]_{\text{max}}$ is so small that laser oscillation can not be expected.

4. Conclusion

For high initial plasma density $(n_{\rm e0} \sim 10^{18} \, {\rm cm^{-3}})$, rapid cooling ($\tau_{\rm E} < 10^{-13} \, {\rm sec}$) can produce laser oscillation of the 164 nm line in a recombining helium plasma. A high one-path gain of $\sim 44 \, {\rm cm^{-1}}$ is obtained for the rapid cooling and lasts for $\sim 10^{-10} \, {\rm sec.}$

For medium initial plasma density $(n_{\rm e0} \sim 10^{16} \, {\rm cm}^{-3})$, the maximum over-population density $[\Delta n_{32}]_{\rm max}$ keeps almost constant for $\tau_{\rm E} \leq 10^{-9}$ sec but is considerably smaller than in the case of $n_{\rm e0} \sim 10^{18} \, {\rm cm}^{-3}$. The gain is estimated to be $\sim 2 \, {\rm cm}^{-1}$ even for $\tau_{\rm E} \sim 10^{-8}$ sec. This indicates that stationary laser oscillation is possible if a sheet plasma flow with a width of more than 5 cm is produced and $T_{\rm e}$ is cooled down with a decay time of 10^{-8} sec. On slower cooling $(\tau_{\rm E} \geq 10^{-7} \, {\rm sec})$, laser oscillation can hardly be realized.

For lower initial plasma densities ($n_{\rm e0} \le 10^{15} \, {\rm cm}^{-3}$), $[\Delta n_{32}]_{\rm max}$ is further reduced and laser oscillation is not possible.

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